# Secondary Measurement

Measurement should be seen as one of the important answers to the question, "Where do numbers come from?" Secondary students develop insights into the essential role of measurement as a link between the abstractions of mathematics and the concreteness of the real world. Most work with measurement connects conceptual development in other strands to practical situations.

Students develop good judgment about using numbers that come from measurement, particularly a realistic sense of what is a meaningful level of precision. Students also develop a critical eye for the validity of measurement; such as, "How well does the instrument measure what it purports to measure?"

Relating to estimation, students must develop a sense for the size of objects using common units.

## *Measurable Performances*

The learner will:

- *Select and utilize appropriate units, tools, and/or technology to achieve the degree of accuracy required in a particular situation.*
- *Investigate the cultural diversity in the development of measurement and summarize the resulting impact.*
- *Utilize measurement on scale models or diagrams to predict and/or validate solutions to problems.*

## *A Closer Look*

Class activities should stress the measurable properties of physical objects, such as length, volume, area, weight, and angle. Measures of time enable the student to study change and rates of change.

Students will investigate how formulas and functions can give rise to compound units, such as meters per second, grams per cubic centimeter, or cycles per second. Students should make sense of converting quantities from one set of units to another, such as meters per second to miles per hour. Students must be given opportunities to apply measurement and estimation skills necessary to become mathematically intelligent consumers. Areas such as building improvement and grocery shopping provide many activities for students.

Students need to develop the skill of thinking in units without conversion, such as metric length, Celsius temperature, or military time. Class activities must give students many repeated experiences with making direct measurements in different systems and estimating using different units. Students should be able to estimate paper length in centimeters, state if it will be dark at 18:00 hours, or decide how to dress if a temperature of 35 degrees Celsius is expected.

Mathematics departments should work with other departments such as science and technology to select a core group of measuring tools and develop a plan for access by all classes and students. Students must become familiar with all types of measuring tools and have opportunities to use the tools repeatedly in real situations.

***Sample Investigation #1***

Investigate the units of measure used in different cultures during different historical time periods. Cultures might include the Chinese, Egyptians, Europeans, Greeks, Babylonians, Arabs, Romans, etc. Find the conversion factors to the present standard system; then make a device or define a method to make a measurement in the ancient unit.

Collect measurement data and estimate the accuracy and precision of the device or method. Investigate life in the time period and suggest reasons for the use of that unit in terms of convenience or accuracy. Find the time span in which the unit was used and try to determine some reasons for changing the unit.

Create a report that summarizes the discoveries. The report should include the historical, precision, and accuracy information with results supported by references or data. Reports can be in different formats depending on ability and experience.

*Notes to Teacher*

- Students could investigate units from their own cultures.

*Extensions*

- Students could investigate units of measurement used in science, such as temperature, mass, and sound. If possible, explore computer interfacing for measurement, such as a temperature probe.

***Sample Investigation #2***

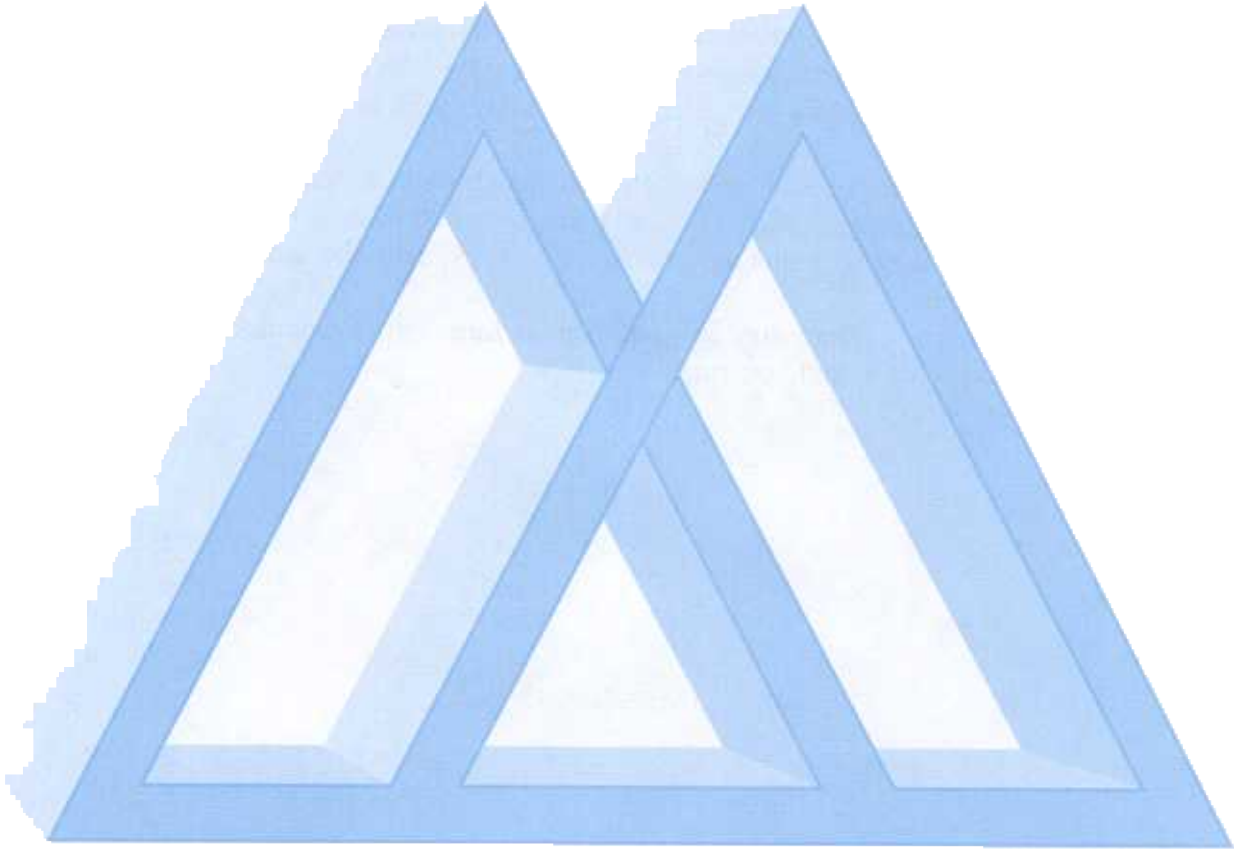
During the school year, many public pools are emptied. Develop some questions that could be answered by taking measurements of the empty pool. With the cooperation of the pool manager, have students make the measurements of the pool, analyze the information, and present or display the results.

*Notes to Teacher*

- Questions students might investigate could include: How much paint would be required to paint the inside of the pool? What volume of water would be needed to fill the pool? How much will the water cost? How long will it take to fill the pool? How much water can be expected to evaporate from the pool during a typical day? What would the chemicals cost to treat the water? How much cement was used to make the pool including the deck area around the pool? How many people can safely swim in the pool?

*Extensions*

- Groups of students could investigate their neighborhood pool and compare the results. Students could make a scale model of the pool, which may be required in some cases to estimate the same measurements.
- Other items to explore may be grain storage bins, fertilizer tanks, railroad tank cars, and box cars.



# Secondary Spatial Relationships/Geometric Topics

Geometry involves various disciplines that permit people to study problems that have intrigued them for centuries. The Egyptians used geometry to social and economic advantage. The Greeks used geometry to measure the circumference of their world. Just as ancient scholars used geometry to understand their world, today, technology enhances the study of geometry and allows all students to investigate their world.

Empowered with geometric tools and ideas, students are able to represent two- and three-dimensional situations. The development of students' reasoning skills in the context of a developing mathematical structure should be included. Students should use inductive reasoning and a variety of methods of proof to create mathematics. The interplay of geometry with algebra allows students to investigate properties of figures, similarities, transformations, and right-triangle trigonometry.

## *Measurable Performances*

The learner will:

- *Investigate and illustrate geometric topics using technology.*
- *Effectively communicate definitions and justify conjectures involving geometric figures.*
- *Use the deductive nature of geometry to solve problems.*
- *Investigate and analyze real situations that allow for algebraic and geometric, as well as other solutions.*
- *Apply spatial sense to two- and three-dimensional situations and effectively communicate the results.*
- *Apply trigonometry to problem situations involving triangles.*

## *A Closer Look*

Work in the spatial relationships/geometric topics strand involves a great deal of interaction and physical activity. In a cooperative-learning environment, students will use tools ranging from compass and straightedge to modern technology to investigate and support conjectures as they build a mathematical structure of their own. The processes of group interaction are as important as the structure they develop.

Activities will be selected that ensure connections of geometry to other disciplines and lets the special talents of individual students draw them into mathematics. Realistic situations, physical models, and the introduction of three-dimensional geometry and two-dimensional representations will enhance the understanding of modeling as a problem-solving tool.

***Sample Investigation #1***

Follow a set of instructions given as intersections of loci from a starting point to a "treasure."

*Notes to Teacher*

- This is an outdoor group activity. Each group must have access to appropriate measuring tools. Each group needs a set of rubber bands of a color different from those of other groups.
- Each treasure spot can be marked with a large nail driven into the ground. As students located the spots, they attach one of their rubber bands to the spot.
- There must be a generous number of treasure spots so that one group will not interfere with the search of another group.
- Students may report their processes in written or oral form. Have students make a scale map of their trail.
- Rewards for success will make the project more memorable

*Extensions*

- Students may generate their own outdoor treasure hunt.

***Sample Investigation #2***

Identify an area, room, or space at home or at school that could be improved by major structural changes. The changes could include rearranging walls, adding furniture, landscaping, or installing new carpet. Start by writing the reasons for the change and the goals to be accomplished as a result of the change. Find the best means of improvement that address the reasons and goals for the change. Record all planning work in a written report that explains the situation, the reasons and goals for the change, the possible solutions, and the final decision. Give a presentation that includes key elements of your report.

*Notes to Teacher*

- Students can make a scale diagram or model of the area to improve.
- Students could propose two or more different ways to address the problem and achieve the goals for change.
- Students could make a scale diagram or scale model of the finished space; they could estimate the cost of the improvement. The cost estimate would include materials and labor required.
- An interior decorator, builder, or construction contractor could be invited to speak with the students and answer questions.
- Students could write the sequence of steps required to complete the project and could estimate the time required.

## Secondary Data Analysis

Using mathematics to explore uncertainties deepens students' data sense. Statistics and probability sharpen the students' power to reason from assumptions and evidence and evaluate confidence in conclusions. Statistics and probability build on collecting, analyzing, representing, and interpreting data.

The construction, analysis, and interpretation of graphs, charts, and tables should be emphasized. Bar graphs, stem and leaf plots, pie charts, histograms, frequency distributions, line plots, tables, matrices, and other methods of displaying and communicating data are essential tools.

Students must recognize misleading representations of data. Students should be given opportunities to support both sides of an issue using the same set of data. In all this work, technology should play a major role. Software, including graphing applications, spreadsheets, data bases, and statistical packages, are basic tools.

Data will be collected by surveys, simulations, or experiments. Students must employ basic ideas of probability including: randomness, independent events, mutually exclusive events, complementary events, equally likely events, and expected value. Probability coupled with statistics allows students to make reliable predictions in real situations, such as insurance, genetics, forecasting, and games.

### *Measurable Performances*

The learner will:

- *Estimate empirical probabilities through the use of simulations and compare with theoretical probability.*
- *Design an experiment to study a problem, conduct the experiment, interpret the results, and communicate the outcomes.*
- *Analyze the effects of data transformations on measures of central tendency and variability.*
- *Apply properties of the normal distribution to data and formulate suitable conclusions.*
- *Verify, interpret, and predict data relationships using graphic utilities and/or statistical software.*

## A Closer Look

Realistic situations, experiments, and games must be used in the study of data analysis. Data from the social sciences and vocational programs, newspapers and magazines can provide interdisciplinary subject matter for data analysis.

Students will use random-sampling methods building a sense for issues, such as bias in sampling, selection of sample size, and nonrepresentative samples.

Students will be able to design and carry out a simple but thorough project using statistical methods. Techniques of data analysis should include measures of central tendency (mean, median, mode), dispersion (range, variance, standard deviation), and relationship (correlation and regression).

Students will find the probability of an event using ideas and methods that include relative frequency, simulation, sample space, tree diagrams, tables, and geometric models. Students will employ counting principles and methods to solve problems that involve combinations, permutations, and Pascal's triangle.

Class activities using experimentation and simulation will construct the binomial distribution. Student-opinion polls for social studies, word or letter counts for English, plant-growth records for biology, and out-of-school activities such as athletics provide opportunities for data analysis with immediate results.

***Sample Investigation***

The National Football League tests all players reporting for training camp for the presence of cocaine, marijuana, and alcohol in their urine. The test will be either positive (drugs are present) or negative (drugs are absent). It is possible to combine several samples as a group and then perform the test on the group to save money.

If the group test result is positive, then individual drug tests will be given on the samples from that group. There are 26 teams in the NFL with 60 players on each team roster for pre-season training camp. If the probability of tested drugs being present is 0.07, what is the most cost-efficient grouping of samples?

*Notes to Teacher*

- Use questions to help students think about the model.
- If the students do not organize their work, encourage the use of a chart.

*Extensions*

- Students familiar with a spreadsheet may see the benefit after performing the calculations repeatedly with a calculator. If not, lead them to the use of a spreadsheet to solve the problem.

- Students familiar with BASIC programming or a programmable calculator may use programming as a method of solution.
- Students could also develop a formula to generalize the process.
- This problem and its solution can be found in the Western Mathematics Scholars Modeling Book and was written by Marla Walz.
- Students could collect data on burning a birthday candle, comparing time and amount of candle remaining. Record as a table or graph and determine an equation.

## Secondary Patterns and Functions

Functions are important unifying ideas in mathematics and other disciplines. Functions allow the representation of relationships among quantities simply and efficiently. The essence of the study of functions involves characterizing the relationships between two quantities and developing a useful sense of the ways in which one quantity can vary in relationship to another. The concept of function is a major building block of secondary mathematics.

Exploring patterns helps students develop mathematical power and instills in them an appreciation for the true beauty of mathematics. The study of patterns requires the students to recognize, describe, generalize, and build mathematical models to predict the behavior of real world phenomena that exhibit the observed pattern.

### *Measurable Performances*

The learner will:

- *Sketch and utilize a graph to estimate the behavior of functions.*
- *Identify a variety of problem situations that can be modeled by the same type of function.*
- *Investigate and illustrate functions and patterns using technology.*
- *Model and analyze real situations using functions and families of functions.*
- *Identify patterns and relate the patterns to functions and the situations they model.*

### A Closer Look

Students should be given problems that allow for development of the tools of algebra. They will define and utilize functions and graphs to visualize the important characteristics of functions. Activities need to be made specific and concrete through discussion of functions and their typical applications.

Technology such as mathematical software packages and graphing calculators must be used to build the basic notion of a function, the relationship and meaning of domain and range for functions, and the comparison and translation among various ways of describing functions. The study of functions needs to proceed with an emphasis on graphical representation, calculator evaluation and

applications, and the ability to form visual images of graphs of functions defined analytically or algebraically.

Students should be encouraged to observe and describe all sorts of patterns in the world around them. Observation of patterns in simple situations can lead to a method of counting applicable to other situations. Students should generate their own problems from actual phenomena.

Sequences and series should be investigated with an emphasis on their descriptions in terms of recurrence relations. Students can use recurrence relations to model real-world phenomena. The Fibonacci sequence can be found in the arrangements of leaves around the branches of trees, or the whorls around a pineapple, which provides an ideal setting for integrating the study of mathematics and botany. Consumer applications such as compound interest, home mortgages, and annuities provide other situations involving recurrence relations.

***Sample Investigation #1***

Do bees build it best? Bees store their honey in honeycombs, which consist of cells they make out of wax. What is the best design for a honeycomb?

*Notes to Teacher*

- Students are led in a discussion on how "best" might be judged.
- Students will discover some basic formulas for calculating the area of specific shapes.
- Students could also use the Pythagorean theorem and trigonometry to find the area of a regular polygon having a fixed perimeter.
- Using the graphing facility on their calculator, students will discover the greater the number of sides the larger the area .

*Extensions*

- Students could consider the volume of prisms with regular polygons as bases. Some questions could include: Which prism has the largest volume for a fixed lateral surface area? Which prism has a base with the largest area for a given base perimeter? Which prism having a regular polygon as a base and tessellates will have the largest volume for fixed base perimeter?

***Sample Investigation #2***

Two high school students were talking about how many children they were going to have when they got older. One of them wanted to have three children. The other was concerned that the world had too many people and planned to have only two children. One said "Why worry about that? Two or three children: it's all about the same." The other said, "Oh, no! It would make a big difference. If everyone had three children instead of two, in just a few generations we would see a VERY big difference!" Who was right?

*Assumptions Developed*

- Assume there are only 1000 newborn babies alive on Earth, 500 are boys, 500 are girls. Everyone else has disappeared.
- Assume that somehow these infants will all grow up without the assistance of anyone.
- Assume everyone gets married on their nineteenth birthday
- Assume every couple has a set of twins or triplets on their twentieth birthday.

*Notes to Teacher*

- It is suggested that the teacher lead the class in a discussion about the use of assumptions that will limit the problem to a manageable size.
- It could be pointed out that population growth models already exist, but in an attempt to be very accurate, they are very complex. They include related factors such as changing life expectancies, food supplies, increasing pollution, technological level of the group being studied, and many more.

*Extensions*

- Students can investigate what effect having other numbers of children per couple has on the outcome. A spreadsheet could be created so the number of children per couple is entered at the top.
- Statistical data on average birth rates within different cultures could be used, then students could predict future population trends.
- Students could add a probability for a couple to have children. Maybe they do not plan to or are not physically able to have children. When calculating the number of couples, multiply by a decimal percent probability of having children. The integer function would need to be used before multiplying to get the number of births.
- Students could also graph the data. This would allow students to see the effect of the different numbers of children per couple on the outcome.

*Extensions*

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- Students could also graph the data. This would allow students to see the effect of the different numbers of children per couple on the outcome.
- Advanced students could derive the formula for the number of persons living. The graphing calculator could also be used to fit an exponential or power curve to the data from the chart or spreadsheet. Students at any level familiar with exponents and the graphing calculator could perform the curve fitting.
- This problem and its solution can be found in the Western Mathematics Scholars Modeling Book and was written by Steve Kaeuper.

## Secondary Algebraic Topics

Algebra is a fundamental tool for thinking and communicating symbolically across many content areas and disciplines. The use of statistics in the natural sciences, economics, psychology, sociology, and other areas is increasing the importance of algebra. As algebra connects to so many other content areas, greater emphasis must be placed on conceptual understanding.

Students use the symbols of algebra to simplify and summarize their understanding of how laws, rules, properties, and definitions relate to arithmetic operations. Mathematical modeling enables students to appreciate the power of mathematics, helping them to understand the world better.

Students will create and interpret graphs, explore their properties, and determine how properties relate to expressions and equations. Since many graphs will be produced by technology, strong connections must be made among tables, graphs, and equations.

### *Measurable Performances*

The learner will:

- *Estimate reasonable solutions of equations.*
- *Develop, communicate, and justify solutions with appropriate algebraic symbols and terminology.*
- *Utilize technology effectively to model real situations.*
- *Investigate and analyze real situations mathematically which allow for multiple solutions, then discuss the appropriateness of each solution.*
- *Develop algebraic properties and recognize the importance of the axiomatic nature of algebra.*

### *A Closer Look*

Important problems in real situations demand a variety of approaches as a check on the reasonableness of the results obtained. Students should be given problems that allow for multiple solutions. Students must have the opportunity to employ multiple methods to solve open-ended problems.

Class activities must use manipulatives to make abstract connections such as the use of algebra tiles to multiply binomials and solve equations. Manipulatives should be used to introduce the concepts with more symbolic

approaches added as reasoning ability develops. Manipulatives used by physical/tactile learners help form a memory web of events and concepts.

Patterns observed in data must be constantly modeled by algebraic methods as well as other methods. Models constructed using spreadsheet software illustrate important uses of the symbols of algebra. Algebraic models may produce inapplicable results; estimating the reasonableness of solutions is an integral part of modeling.

Use of graphing utilities must increase as the students' conceptual understanding develops. Along with the symbolic solution to a system of equations, students should estimate the solution using a graphing utility.

***Sample Investigation #1***

Select an intersection and determine how long the yellow light of a traffic light or traffic signal should be on.

*Notes to Teacher*

- Students at this level relate well to the problem since most are new drivers or soon will be.
- Students will use data on reaction time and stopping distances at different speeds, which may be available in drivers' manuals.
- Students will need to measure street widths and widths of pedestrian lanes.
- Student use of physical measurement is an important part of this investigation.
- Students may also get street information from county/city planners.
- Students working on the same problem can network with another group of students to compare processes and results. Urban/rural or international comparisons would be interesting.

*Extensions*

- Use technology to analyze data, model, and report results (e.g., spreadsheets, or graphing utilities).
- Use telecommunications to share data with a collaborating group.
- In a city, students could focus on "high accident" intersections.

***Sample Investigation #2***

Students are asked to collect formulas from different areas, such as science, medicine, agriculture, engineering, business, finance, automotive repair, manufacturing, construction, fine art, photography, etc. Students would describe a few problems that illustrate the use of the formula. They should be sure to show the steps in solving the formula. Graphics or pictures could be found relating to the situation and students could make displays of the formulas. The different formulas could be displayed in the classroom or hallway.

## Secondary Discrete Mathematics

Discrete mathematics is a relatively new area in which a situation may be represented with models having distinct and countable elements, as opposed to continuous. The nature of discrete mathematics and digital computers promises considerable mathematical empowerment to any who can master both.

Finite graphs and their matrix representations are important additions to a student's repertoire of problem-solving tools. The diagrams, networks, and flowcharts that students construct to model situations or use for planning, scheduling, and decision-making are explored for their mathematical properties.

Students will experience developing and analyzing algorithms in a variety of situations. They will work with both sequential and repeating algorithms. Students will create and compare algorithms making decisions about wher certain algorithms work best.

### *Measurable Performances*

The learner will:

- *Develop and analyze algorithms.*
- *Represent and analyze finite graphs using matrices.*
- *Utilize technology effectively to represent and analyze real situations represented by discrete models.*
- *Apply enumeration and finite probability to real situations and effectively communicate the results.*
- *Formulate a discrete mathematics problem, model it, and effectively report the results.*

### *A Closer Look*

Students will explore naturally occurring situations such as scheduling, routing, and networking by representing them with discrete models. When models involve graphs, analysis may be performed using matrices. Matrices will be manipulated using graphic calculators and computer software.

In real situations, students will employ techniques such as critical-path methods, hierarchies, cross-classifications, and recursion to reduce complicated situations to simpler ones. Students will construct, analyze, and compare algorithms, and use graphical methods of linear programming and their discrete analogues as decision-making techniques.

A cooperative-learning environment is important to discrete mathematics processes because individuals who may use this type of mathematics professionally will likely be working as part of a group.

Students will communicate the results of investigations using discrete models.

***Sample Investigation #1***

The steps necessary for the departure of a passenger aircraft once it has landed, the time needed for the completion of each step, and the prerequisite steps are given in the following table.

| | Task | Time in Minutes | Prerequisite Steps |
|---|---|---|---|
| A | passengers disembark | 12 | plane has landed |
| B | fuel the aircraft | 23 | plane has landed |
| C | remove baggage | 15 | plane has landed |
| D | clean interior | 10 | A |
| E | load baggage | 12 | C |
| F | load passengers | 14 | D |
| G | restock kitchen | 4 | A, C |
| H | gain taxi clearance | 7 | E, F, G |
| I | seat belt check | 2 | H |
| J | taxi to runway | 6 | H |
| K | take-off clearance | 2 | J |

Construct a diagram showing times for completion of each task and precedence. Determine the critical path.

*Notes to Teacher*

- The data in the above table could be gathered by the students for other types of jobs.
- Along with the diagram, have students explain in oral or written form how they found the critical path.
- Have students select a real situation and model it with a diagram for the purpose of determining the critical path.

***Sample Investigation #2***

The Stick-Em Bubble Gum Company decides to promote its gum by including in each pack the photo of one of five rookies from the 1933 baseball season. Assuming that there are equal numbers of photos of each

of the five rookies and that when you buy a pack of gum your chances of getting any of the five photos are the same, about how many packs of gum would you expect to have to buy to get all five photos?

*Notes to Teacher*

- After they have read and understand the problem described above, ask students to make some guesses.
- Questions for discussion:
  1. About how many packs would you expect to have to buy?
  2. Is it possible to get all five photos with only five packs? Would you expect that to happen?
  3. Is it possible not to get all five photos with 100 packs? Would you expect that to happen?
  4. Do you think the problem is realistic?
  5. Has anyone had an experience collecting objects in offers like the one described?
  6. If you knew that you would need to buy about 100 packs of gum to obtain all five cards, would you still attempt to do it?
- Model the problem by running a simulation; then collect class data.
- Use the data to estimate probabilities related to the discussion.

*Extensions*

- Look back and discuss ways to improve the estimate
- Define a similar problem; use simulation to explore the problem

*Some Discrete Mathematics Topics*

- Algebraic Systems
- Boolean Algebra
- Combinatorial analysis
- Critical path Analysis, PERT (Program Evaluation and Review Technique)
- Directed graphs
- Finite state machines
- Functions
- Graph theory
- Knapsack problems
- Matching Problems
- Mathematical Induction
- Matrices
- Network Analysis
- Networks
- Planar Graphs, Graph Coloration
- Posets (partially ordered sets) and Lattices
- Propositional Calculus
- Recurrence Relations
- Relations
- Sets
- Sorting and Searching Algorithms
- The Pigeonhole Principle
- Trees
- Vectors

## Secondary Advanced Topics

Upon completion of the mathematics curriculum, students should be able to enter any college mathematics course with confidence. Advanced topics are enrichment for the college-bound student.

Students who have acquired a foundation across a broad mathematics curriculum are ready to examine mathematical structure in more detail. They can appreciate that seemingly different mathematical systems may be essentially the same. They can investigate the properties of functions and analyze the real number system and its various subsystems with regard to their structural characteristics.

Students can recognize the capabilities and limitations of current technology and can use technology in independent investigations of mathematics and science.

Students can model real world phenomena using circular, exponential, and logarithmic functions. Students will apply this knowledge to solve equations and verify identities.

Students are prepared for a formal treatment of limits which may be followed with fundamental definitions and applications of differential and integral calculus.

Detailed development of experiments involving data sampling would extend the work from previous levels. Students would apply many different data distributions and perform calculations of sample size for required confidence levels.

### *Measurable Performances*

The learner will:

- *Use the definition of a derivative to effectively examine the properties of a function.*
- *Identify and evaluate the capabilities and limitations of technology as applied to a problem situation.*
- *Effectively apply formal definitions that form the basis of calculus to various functions.*
- *Verify group and field properties and effectively communicate the results.*

- *Identify real situations in which the methods of calculus are useful.*
- *Design an experiment to study a problem, conduct the experiment, interpret the results, and communicate the outcomes.*
- *Apply properties of data distributions to data and formulate suitable conclusions.*
- *Investigate and apply the connection between trigonometric and circular functions.*
- *Explore and apply logarithmic and exponential functions.*

## A Closer Look

Students will work with a minimum of teacher direction. They will use sophisticated mathematics software such as numeric and symbolic processors, electronic textbooks, and/or computer programming languages to explore mathematics.

Students will identify real situations to which elements of calculus and trigonometry may be applied. Cooperative groups will then solve the problem and communicate the results.

Students will communicate mathematics with clarity, precision, and mathematical correctness in oral and written forms. They will use modern communication technology and physical models of their own making to aid in concept communication.

***Sample Investigation #1***

Students will choose a real problem that may be treated with calculus. They will produce a video presentation based on this problem to promote the importance of calculus.

*Notes to Teacher*

- This investigation is independent of teacher direction. The teacher should aid in locating resources and equipment.
- In an Advance Placement course, this is a good project to be performed as a culminating project after the AP test and before the end of school.
- Be sure students have access to appropriate video equipment.

***Sample Investigation #2***

Find data that is periodic, such as sunrise times on the first day of the month, size of the moon each day throughout the month, the height of a chair on a ferris wheel, etc. Use the sine and cosine functions to model the data.

*Notes to Teacher*

- Estimated data for the ferris wheel could be collected from a video tape. A video player that can display a clear still frame would allow for fairly accurate results.
- Students can use a graphing utility to plot both the data and the equation used as a model. Students could use statistical methods to determine the goodness of fit.